NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION : BACHELOR OF SCIENCE APPLIED MATHEMATICS AND STATISTICS	
QUALIFICATION CODE: 07BSAM	LEVEL: 5
COURSE CODE: PBT501S	COURSE NAME: PROBABILITY THEORY 1
SESSION: JULY 2023	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

SUPPLEMENTARY/ SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	Dr. D. Ntirampeba Mr. E. Mwahi
MODERATOR:	Mr. A. Roux

THIS QUESTION PAPER CONSISTS OF 4 PAGES
(Excluding the cover page)

INSTRUCTIONS

1. Answer ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

1. Non-programable calculator

ATTACHMENTS
1. Statistical tables (Z-Table)

Question 1 [25 marks]

1.1

Assume that $\{B_1, B_2, ..., B_k\}$ is a partition of S such that $P(B_i) > 0$, for $i = 1, 2, ..., k$.

Prove that for any event A in the sample S, $P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$ [5]

1.2

Let X be a binomial random with a probability mass function given by

$$f(x) = \begin{cases} \binom{n}{x} p^k q^{n-x}, & for\ x = 0, 1, ..., n \\ 0, & elsewhere \end{cases}$$

Show that $E(X) = np$. [10]

1.3

Dr.Richmond, a psychologist, is studying the daytime television viewing habits of college students. She believes 45 percent of college students watch soap operas during the afternoon. To further investigate, she selects a sample of 10.

1.3.1 Write down a probability distribution for the number of students in the sample who watch soap operas. [3]

1.3.2 Find the mean and variance of this distribution. [4]

1.3.3 What is the probability of finding exactly four watch soap operas? [3]

Question 2[30 marks]

2.1 Consider the experiment of tossing a fair coin three times.

2.1.1 Develop a tree diagram for the experiment. [2]

2.1.2 List the experimental outcomes. [8]

2.1.3 What is the probability for each experimental outcome? [1]

2.2. A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways can this be done when:

2.2.1 at least 2 women are included? [4]

2.2.2 at most 2 women are included? [4]

2.3

The following information is based on the trends in the United States published by the Food Marketing Institute, Washington, D.C. The columns represent length of customer loyalty (in years) at a primary supermarket. The rows represent regions in United States

| Region | Loyalty | | | | | | Row total |
	Less than a ye1-ar	1-2 years	3-4 years	5-9 years	10-14 years	15 or more years	
East	32	54	59	112	77	118	452
Midwest	31	68	68	120	63	173	523
South	53	92	93	158	106	158	660
West	41	56	67	78	45	86	373
Column total	157	270	287	468	291	535	2008

What is the probability that a customer chosen at random

2.3.1   has been loyal at least 10 years or is from Midwest?   [2]
2.3.2   has been loyal at least 10 years, given that he or she is from South or West?   [3]
2.4.

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

2.4.1. If a finished product is randomly selected and is found to be defective, what is the
        probability that it was made by machine B_2?   [4]

2.4.2. If a finished product is randomly selected and is found to be defective, what is the
        probability that it was made by machines B_1 and B_2?   [2]

2

Question 3 [20 marks]

3.1. The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by

X	0	1	2	3	4
p(x)	0.41	0.37	0.16	0.05	0.01

3.1.1. What is the probability of no imperfections in 10 meters of a synthetic fabric? [1]

3.1.2. What is the probability that there are at least three imperfections in 10 meters of a synthetic fabric? [2]

3.1.3. What is the expected number of imperfections in 10 meters of a synthetic fabric? [3]

3.1.4. What is the standard deviation of the imperfections in 10 meters of a synthetic fabric? [3]

3.1.5. Compute the coefficient of variation for the imperfections in 10 meters of a synthetic fabric? [2]

3.1.6. Construct the cumulative distribution of X and hence find the median of X [4]

3.2 A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyshev's theorem to estimate $P(|X - 10| \geq 3)$ [5]

Question 4 [25 marks]

4.1 Let X be random variable with a probability mass function given by

$$f(x) = \begin{cases} \dfrac{e^{-\mu}\mu^x}{x!}, & for\ x = 0,1,2\ ..., \\ 0, & elsewhere \end{cases}$$

Show that $\sum_{x=0}^{+\infty} f(x) = 1$ [5]

4.2 The number of typing errors made by a typist has Poisson distribution with an average of four errors per page. If more than four errors on a given page, the typist must retype the whole page.

4.2.1 What is the probability that a certain page does not have to be retyped? [5]

4.2.2 What is the expected number of typing errors will the typist make in 5 pages? [2]

4.2.3   What is the standard deviation number of typing errors will the typist make in 5 pages

[3]

4.3

University and college students average 7.2 hours of sleep per night, with a standard deviation of 40 minutes. If the amount of sleep is normally distributed,

4.3.1   What proportion of university and college students sleep for more than 8 hours? [5]

4.3.2   Find the amount of sleep that is exceeded by only 25% of students.   [5]
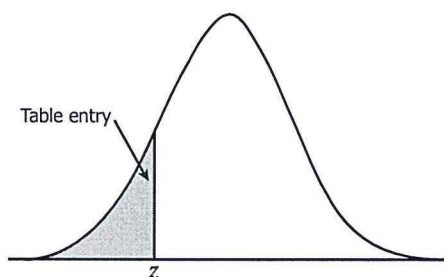
Standard Normal Probabilities

Table entry

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

Table entry

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998